AFFINGE GROUP SCHEMES 12

DEFINITION 0.1. A **flag** of subspaces in $V = \mathbb{k}^n$ is a sequence

$$\{0 \subsetneq V_1 \subsetneq V_2 \subsetneq \cdots \subsetneq V_n = V\}$$

The set of all flags of subspaces in V is parametrized by a variety $\mathrm{Flag}(\mathbb{k}^n)$ called **the flag variety of** V.

DEFINITION 0.2. For $g = (\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n) \in M_n(\mathbb{k})$, We define its associated flag as

$$\mathrm{flag}(g) = \{0 \subsetneq \langle \mathbb{v}_1 \rangle \subsetneq \langle \mathbb{v}_1, \mathbb{v}_2 \rangle \subsetneq \cdots \subsetneq \langle \mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n \rangle = V\}$$

$\mathrm{GL}_n(\mathbb{k})$ acts on $\mathrm{Flag}(\mathbb{k}^n)$:

$$A.\,\mathrm{flag}(g) = \mathrm{flag}(Ag).$$

PROPOSITION 0.3. *Let us denote by B the set of upper triangular matices. Then:*

$$\mathrm{Flag}(\mathbb{k}^n) \cong \mathrm{GL}_n(\mathbb{k})/B$$

PROPOSITION 0.4. $\mathrm{GL}_n(\mathbb{k}) = \coprod_{w \in \mathfrak{S}_n} BwB$