RESOLUTIONS OF SINGULARITIES.

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08. Coefficient ideal

From the paper of Encinas and Hauser:

. . . .

The coefficient ideal of an ideal I of W at a with respect to V is an ideal in V which is built from the coefficients of the Taylor expansion of the elements of I with respect to the equations defining V. Let x, y and y be regular systems of parameters of $\mathcal{O}_{W,a}$ and $\mathcal{O}_{V,a}$ so that $x = 0$ defines V in W. For f in I denote by $a_{f,\alpha}$ the elements of $\mathcal{O}_{V,a}$ so that $f = \sum_\alpha a_{f,\alpha} \cdot x^\alpha$ holds after passage to the completion. Then we set

$$\operatorname{coeff}_V I = \sum_{|\alpha|<c} (a_{f,\alpha}; f \in I)^{\frac{c!}{c-|\alpha|}}.$$

. . . .

Example 8.1. Let us consider a curve $C = \{(t^3, t^5, t^7)\} \in \mathbb{A}^3$. By using a theory of groebner basis, we may eliminate the variable t and obtain an ideal

$$I = (y^7 - z^5, xz - y^2, xy^5 - z^4, x^2y^3 - z^3, x^3y - z^2, x^4 - zy)$$

Let us choose $V = \{z = 0\}$ as the hypersurface. Then:

(1) When $c = 1$, we have
$$\operatorname{coeff}_V(I) = (y^2, x^3y, x^4)$$
(2) When $c = 2$, we have
$$\operatorname{coeff}_V(I) = (y^2, x^3y, x^4) + (x, y)^2 = (x^2, xy, y^2)$$
(3) When $c = 3$, we have
$$\operatorname{coeff}_V(I) = (y^2, x^3y, x^4)^2 + (x, y)^3 + (1)^6 = (1).$$

Problem 8.2. To compute the coefficient ideal of a given ideal I, Is it sufficient to consider only coefficients of generators of I?

Let X be a closed subscheme of a regular scheme W. We want to resolve the singularity of X. If there exists an regular hypersurface V such that $V \supset X$, then we may replace W by V. So we may (have to) assume that X is not contained in such hypersurfaces. Instead, we have for each point $a \in X$ a "hypersurface of maximal contact" V. V is not canonical, but is good enough to define the invariant i_a and then (afterwards) determine the center of blow up.