APPENDIX: LOCAL APPEARANCE OF BLOW UP

Let us define a ring homomorphism φ as follows.

$$\begin{array}{ccc} \varphi : A[X_0, \dots, X_s] & \to & A[f_0 \cdot T, \dots f_s \cdot T, (f_j \cdot T)^{-1}]_0 \\ \cup & & \cup \\ X_i & \mapsto & (f_i \cdot T)(f_j \cdot T)^{-1} \end{array}$$

It is easy to see that φ is a surjective homomorphism.

$$\begin{aligned} & p(x_0, x_1, \dots x_s) \in \operatorname{Ker}(\bar{\varphi}) \\ \Longleftrightarrow\ & p\left(\frac{f_0 T}{f_j T}, \dots, \frac{f_s T}{f_j T}\right) = 0 \\ \Longleftrightarrow\ & \sum_{i_0, i_1, \dots i_s} p_{i_0 i_1 \dots i_s} \left(\frac{f_0 T}{f_j T}\right)^{i_0} \cdot \cdots \cdot \left(\frac{f_s T}{f_j T}\right)^{i_s} = 0 \\ \Longleftrightarrow\ & \exists N > 0 \quad \sum_{i_0, i_1, \dots i_s} p_{i_0 i_1 \dots i_s} (f_0 T)^{i_0} \cdot \cdots \cdot (f_s T)^{i_s} (f_j T)^{N - (i_0 + i_1 + \cdots + i_s)} = 0 \\ \Longleftrightarrow\ & \sum_{i_0, i_1, \dots i_s} p_{i_0 i_1 \dots i_s} f_0^{i_0} \cdot \cdots \cdot f_s^{i_s} f_j^{N - (i_0 + i_1 + \cdots + i_s)} = 0 \\ \Longleftrightarrow\ & p(f_0 f_j^{-1}, \dots f_s f_j^{-1}) = 0 \quad \text{ in } A[f_j^{-1}]. \end{aligned}$$

We conclude:

PROPOSITION 0.1. *Let us denote by* $A[f_0 f_j^{-1}, \dots f_s f_j^{-1}]$ *the subalgebra of* $A[f_j^{-1}]$ *generated by* $f_0 f_j^{-1}, \dots f_s f_j^{-1}$*. (Note that this notation is ambiguous and should not be used without an explanation.) Then the ring homomorphism* φ *as above induces an algebra isomorphism*

$$\bar{\varphi} : A[f_0 f_j^{-1}, \dots f_s f_j^{-1}] \cong A[f_0 T, \dots f_s T, (f_j T)^{-1}]_0.$$