ZETA FUNCTIONS. NO.7

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Congruent zeta as a zeta of a dynamical system.

The definition of Artin Mazur zeta function is valid without assuming the number of the base space M to be a finite set.

DEFINITION 7.1. Let M be a set. Let $f : M \to M$ be a map such that $\# \operatorname{Fix}(f^n)$ is finite for any $n > 0$. We define the Artin-Mazur zeta function of a dynamical system (M, f) as

$$Z((M,f),T) = \exp(\sum_{j=1}^{\infty} \frac{\# \operatorname{Fix}(f^j) T^j}{j})$$

Let q be a power of a prime p. We may consider an automorphism Frob_q of $\bar{\mathbb{F}}_q$ over $\mathbb{F}_q$ by

$$\operatorname{Frob}_q(x) = x^q$$

PROPOSITION 7.2. $\operatorname{Frob}_q : \mathbb{F}_{q^r} \to \mathbb{F}_{q^r}$ *is an automorphism of order* r*. It is a generator of the Galois group* $\operatorname{Gal}(\mathbb{F}_{q^r}/\mathbb{F}_q)$*.*

For any projective variety X defined over $\mathbb{F}_q$, we may define a Frobenius action Frob_q on $X(\bar{\mathbb{F}}_q)$:

$$\operatorname{Frob}_q([x_0 : x_1 : \dots x_N]) = ([x_0^q : x_1^q : \dots x_N^q]).$$

For any $\bar{\mathbb{F}}_q$-valued point $x \in X(\bar{\mathbb{F}}_q)$, We have

$$\operatorname{Frob}_q^r(x) = x \iff x \in X(\mathbb{F}_{q^r}).$$

PROPOSITION 7.3. *The Artin Mazur zeta function of the dynamical system* $(X(\bar{\mathbb{F}}_q), \operatorname{Frob}_q)$ *conincides with the congruent zeta function* $Z(X/\mathbb{F}_q, t)$*.*