ZETA FUNCTIONS. NO.6

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Let f be a self map $f : M \to M$ of a set M. It defines a (discrete) **dynamical system**.

To explain the basic idea, we first examine the case where M is a finite set.

We put $A = C(M, \mathbb{C})$, the set of $\mathbb{C}$-valued functions on M.

f defines a pull-back of functions:

$$f^*(a)(x) = a(f(x)) \qquad (a \in A)$$

and push-forward:

$$f_*(a)(x) = \sum_{f(y)=x} a(y) \qquad (a \in A).$$

(It might be better to treat the push-forward as above as a push-forward of measures.)

We note also that any element of A admits an integration

$$\int_M a = \sum_{x \in M} a(x) \qquad (a \in A)$$

(which is a integration with respect to the counting measure.)

Proposition 6.1. *We have*

$$\int_M (f^* a) b = \int_M a(f_* b)$$

In other words, f_* is the adjoint of f^*.

Definition 6.2. We define the set $\mathrm{Fix}(f)$ as the set of fixed points of f. Namely,

$$\mathrm{Fix}(f) = \{x \in M; f(x) = x\}.$$

Proposition 6.3. $\mathrm{tr}(f^*) = \mathrm{tr}(f_*) = \#\,\mathrm{Fix}(f)$.

Definition 6.4. We define the Artin-Mazur zeta function of a dynamical system (M, f) as

$$Z((M, f), T) = \exp(\sum_{j=1}^{\infty} \frac{\#\,\mathrm{Fix}(f^j) T^j}{j})$$

Proposition 6.5.

$$Z((M, f), T) = \frac{1}{\det(1 - T f^*)}$$